Enskog-Landau kinetic equation. Calculation of the transport coefficients for charged hard spheres

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Abstract

Using charged hard spheres model as an example, the dense one-component plasma is considered. For this model the Enskog-Landau kinetic equation is obtained and its normal solution is found using Chapman-Enskog method. Transport coefficients are obtained numerically and analytically and compared with the experimental data available.
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Construction of kinetic equations for dense gases and plasma is one of the most important problem in the kinetic theory of classical systems. A consequent approach for construction of kinetic equations has been done by Bogolubov [1]. This approach is based on a chain of equations for s-particle distribution functions and on boundary conditions for weakening correlations. Using such boundary conditions, we can in principle, express all s-particle distribution functions in terms of the single-particle function and obtain for it a closed kinetic equation. There is a large number of approaches for derivation of kinetic equations [2, 3, 4, 5]. Despite a difference of these approaches between themselves in shape, the weakening correlation principle in one or another form has been used in all approaches just as in Bogolubov's method. However, all these approaches are most efficient in the case when a small parameter (density, interaction, etc.) is present. For dense gases and dense plasma small parameters are absent. In this case analysis of the BBGKY hierarchy becomes very difficult because we can not restrict ourselves to some finite number of terms in expansion for the collision integral. Moreover, an additional problem, concerning correct account in the collision integrals of interactions between particles on short as well as long distances, arises.

Relatively recently an approach, based on a modification of the weakening correlation principle, has been proposed [6]. This approach leads to a consequent construction of kinetic equations for dense gases without additional phenomenological assumptions. New boundary condition to the BBGKY hierarchy take into account a non-equilibriumnes of single particle distribution function as well as local conservation laws of mass, momentum and energy, i.e., the quantities which constitute the basic for the hydrodynamic description of evolution of the system. In the "pair collision" approximation, such approach leads to an Enskog-like kinetic equation. Similar ideas have been proposed independently by Karkheck, van Beijeren, de Schepper and Stell [7] at derivation of the kinetic equation for the "square-well" potential. Somewhat different modification of Bogolubov's approach has been considered by Rudyak [8, 9]. Here, the Enskog-like kinetic equation for a system of hard spheres has been obtained and attempts to extend this equation on a system with soft potential have been made.

The ideas of work [6], which is based on Zubarev's non-equilibrium statistical operator method [10, 11], stimulate a revision of the problem connected with constructing of kinetic equations for dense gases and plasma. Investigations [12, 13] were logical continuation of the work [6] in which original result has been achieved: a consequent derivation of the kinetic equation of revised Enskog theory [14, 15, 16] for a system of hard spheres.

In the present paper a kinetic equation for the single-particle distribution function is obtained from the BBGKY hierarchy with modified boundary condition in the "pair collision" approximation. This kinetic equation is valid for moderately dense classical systems with the interparticle potential in a form of hard sphere potential plus some long-range potential $\Phi^l(r)$. In the case when $\Phi^l(r)$ is the Coulomb potential, we have obtained a kinetic equation, called Enskog-Landau one, for a system of charged hard spheres. Normal solutions of this equation are found by the Chapman-Enskog method. On the basis of the solutions, analytical expressions for viscosity coefficients and thermal conductivity are obtained. Numerical calculations of transport coefficients are performed for neutral and ionized argon. The results are presented in a temperature dependent form. A comparison between theoretically predicted values for transport coefficient and experimental data is examined.

The BBGKY hierarchy of equations for non-equilibrium distribution functions of classical interacting particles has been obtained in the paper [6] on the basis of assembling time retarded solutions for Liouville equation with modified Bogolubov's condition meaning weakening correlations between particles. According to Zubarev's non-equilibrium statistical operator method [10, 11], full non-equilibrium distribution function $\rho\left(x^N;t\right)$ for all N particles of the system satisfies the following asymptotic condition:

$$\lim_{t_0\to-\infty}\exp\left(\imath L_N t_0\right)\left(\rho\left(x^N,t_0\right)-\rho_q\left(x^N,t_0\right)\right)=0. \tag{1}$$

Here, the limit $t_0\to-\infty$ is made after thermodynamical one $N\to\infty, V\to\infty$, $N/V\to$ const, $\imath=\sqrt{-1}$ and L_N is the Liouville operator:

$$L_N=\sum_{j=1}^{N}L\left(j\right)+\frac{1}{2}\sum_{\substack{j=1\\ j\neq k}}^{N}\sum_{k=1}^{N}L\left(j,k\right),$$

$$L\left(j\right)=-\imath\frac{\boldsymbol{p}_j}{2m}\frac{\partial}{\partial\boldsymbol{r}_j},\qquad L\left(j,k\right)=\imath\frac{\partial\Phi\left(|\boldsymbol{r}_{jk}|\right)}{\partial\boldsymbol{r}_{jk}}\left(\frac{\partial}{\partial\boldsymbol{p}_j}-\frac{\partial}{\partial\boldsymbol{p}_k}\right), \tag{2}$$

Φ_{jk} is the interaction energy between two particles j and k; $x_j=\{\boldsymbol{r},\boldsymbol{p}\}$ is the set of phase variables (coordinates and momenta). Quasi-equilibrium distribution function $\rho_q\left(x^N;t\right)$ is determined from the condition of maximum for informational entropy at fixed values of the single particle distribution function $f_1\left(x_1;t\right)$ and average density of the interaction energy $\langle\mathcal{E}_{int}\left(\boldsymbol{r}\right)\rangle^t$, $\left(\langle\ldots\rangle^t=\int d\Gamma_N\ldots\times\ \rho\left(x^N;t\right),\ d\Gamma_N=\frac{\left(dx\right)^N}{N!}\right)$, that corresponds to taking into account correlations, related to conservations laws of hydrodynamical variables for particle density $n\left(\boldsymbol{r};t\right)$, momentum $\boldsymbol{j}\left(\boldsymbol{r};t\right)$ and full energy $\mathcal{E}\left(\boldsymbol{r};t\right)$ [17]. This function can be presented as follows [6, 13]:

$$\rho_q\left(x^N,t\right)=\exp\left(-U_N\left(\boldsymbol{r}^N;t\right)\right)\prod_{j=1}^{N}\frac{f_1\left(x_j;t\right)}{u\left(\boldsymbol{r}_j;t\right)}, \tag{3}$$

where $u\left(\boldsymbol{r}_j;t\right)$ is obtained from the relations:

$$u\left(\boldsymbol{r}_1;t\right)=\int\frac{d\boldsymbol{r}^{N-1}}{(N-1)!}\exp\left(-U_N\left(\boldsymbol{r}_1,\boldsymbol{r}^{N-1};t\right)\right)\prod_{j=2}^{N}\frac{n\left(\boldsymbol{r}_j;t\right)}{u\left(\boldsymbol{r}_j;t\right)},$$

$$U_N\left(\boldsymbol{r}^N;t\right)=\sum_{j<k}^{N}\Phi_{jk}\beta_{jk},\qquad \beta_{jk}=\beta\left(\boldsymbol{r}_j,\boldsymbol{r}_k;t\right)\equiv\frac{1}{2}\Big(\beta\left(\boldsymbol{r}_j;t\right)+\beta\left(\boldsymbol{r}_k;t\right)\Big),$$

$n\left(\boldsymbol{r};t\right)=\int d\boldsymbol{p}f_1(x;t)$ is non-equilibrium particles concentration, β is certain function, being an analogue of local inverse temperature.

Taking into account the boundary condition (1) is equivalent to the transition from the Liouville equation to a modified one [10, 11]:

$$\left(\frac{\partial}{\partial t}+\imath L_N\right)\rho\left(x^N,t\right)=-\varepsilon\Big(\rho\left(x^N,t\right)-\rho_q\left(x^N,t\right)\Big). \tag{4}$$

This equation contains the small source in the right-hand side, which destroys the invariance with respect to time inversion ($\varepsilon\to+0$ after the thermodynamic limit transition).

chain for the s-particle non-equilibrium distribution function $f_s\left(x^s;t\right)=\int d\Gamma_{N-s}\rho\left(x^N;t\right)$ [6, 13]:

$$\left(\frac{\partial}{\partial t}+\imath L_s\right)f_s\left(x^s;t\right)+\sum_{j=1}^{s}\int dx_{s+1}\,\imath L\left(j,s+1\right)f_{s+1}\left(x^{s+1};t\right)=$$

$$=-\varepsilon\left(f_s\left(x^s;t\right)-g_s\left(\boldsymbol{r}^s;t\right)\prod_{j=1}^{s}f_1\left(x_j;t\right)\right), \tag{5}$$

where

$$g_s\left(\boldsymbol{r}^s;t\right)=\int d\Gamma_{N-s}\,d\boldsymbol{p}^s\,\rho_q\left(x^N,t\right) \tag{6}$$

is the quasi-equilibrium s-particle coordinate distribution function which depends on $n\left(\boldsymbol{r};t\right)$ and $\beta\left(\boldsymbol{r};t\right)$ functionally. Due to the fact, that $g_1\left(\boldsymbol{r}_1;t\right)=1$, the equation chain (5) is distinguished from the ordinary BBGKY hierarchy [1] by the availability of sources in the right-hand parts of the equations beginning from the second one and takes into account both the one-particle and collective hydrodynamical effects.

Let us consider the solution to equation chain (5) within the pair collision approximation. In this case, three- and higher-particle correlations are neglected, but environment influence to the evolution of a chosen pair of particles is taken into consideration by means of application of the boundary condition. Then from (5) for $f_2\left(x_1,x_2;t\right)$ we obtain [6]:

$$\left(\frac{\partial}{\partial t}+\imath L_2+\varepsilon\right)f_2\left(x_1,x_2;t\right)=\varepsilon g_2\left(\boldsymbol{r}_1,\boldsymbol{r}_2;t\right)f_1\left(x_1;t\right)f_1\left(x_2;t\right). \tag{7}$$

The formal solution of equation (7) is of the form

$$f_2\left(x_1,x_2;t\right)=$$
$$=\varepsilon\int_{-\infty}^{0}d\tau\exp\Big(\left(\varepsilon+\imath L_2\right)\tau\Big)g_2\left(\boldsymbol{r}_1,\boldsymbol{r}_2;t+\tau\right)f_1\left(x_1;t+\tau\right)f_1\left(x_2;t+\tau\right). \tag{8}$$

Substituting (8) into (5) at $s=1$ leads to the kinetic equation for one-particle distribution function in the pair collision approximation:

$$\left(\frac{\partial}{\partial t}+\imath L(1)\right)f_1\left(x_1;t\right)=I_{col}\left(x_1;t\right), \tag{9}$$

where

$$I_{col}\left(x_1;t\right)=-\int dx_2\imath L(1,2)\varepsilon\int_{-\infty}^{0}d\tau\exp\Big(\left(\varepsilon+\imath L_2\right)\tau\Big)F_2\left(x_1;t+\tau\right), \tag{10}$$

$$F_2\left(x_1;t+\tau\right)=g_2\left(\boldsymbol{r}_1,\boldsymbol{r}_2;t+\tau\right)f_1\left(x_1;t+\tau\right)f_1\left(x_2;t+\tau\right)$$

is the collision integral. We must emphasize that equation (9) is needed to be adjusted with the equation for binary quasi-equilibrium distribution function $g_2\left(\boldsymbol{r}_1,\boldsymbol{r}_2;t\right)$. According to (3) and (6) this function functionally depends on $n\left(\boldsymbol{r};t\right)$, $\hat{\mathcal{E}}_{int}\left(\boldsymbol{r};t\right)$ (or on $\beta\left(\boldsymbol{r};t\right)$). Besides it was shown, that quasi-equilibrium correlation distribution function $h_2\left(\boldsymbol{r}_1,\boldsymbol{r}_2;t\right)$, which is related to $g_2\left(\boldsymbol{r}_1,\boldsymbol{r}_2;t\right)$ $\left(h_2=g_2-1\right)$, satisfies the Ornstein-Zernike equation [12]. In paper [18] a non-equilibrium grand canonical distribution for the system of hard spheres and on the base of methods of non-equilibrium statistical mechanics [19] an Ornstein-Zernike equation for pair quasi-equilibrium correlation function of hard spheres

paper [12]. New equation for $h_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t)$ is an analogue of Ornstein-Zernike equation in equilibrium statistical mechanics [19]. This equation for the hard sphere system in equilibrium case has exact solution in Percus-Yevick approximation [12].

In the paper [6] some special cases were considered, when the interparticle interaction potential Φ_{kj} is modelled as the hard sphere potential $\Phi^{hs}(|\boldsymbol{r}|)$ for particles with diameter σ. Then taking into account the singularity of the hard sphere potential ($\tau \to +0$, τ is collision time) it was shown for the first time how the collision integral (10) transforms into collision integral of revised Enskog theory (RET) [14]:

$$I_{col}^{hs}(x_1; t) = \int dx_2 \, \hat{T}^{hs}(1,2) g_2^{hs}(\boldsymbol{r}_1, \boldsymbol{r}_2; t) \, f_1(x_1; t) \, f_1(x_2; t), \tag{11}$$

where $\hat{T}^{hs}(1,2)$ is the Enskog's collision operator for hard spheres, $g_2^{hs}(\boldsymbol{r}_1, \boldsymbol{r}_2; t)$ is the pair quasiequilibrium distribution function of hard spheres, which depends on the average non-equilibrium density $n(\boldsymbol{r}; t)$ functionally. It is important to emphasize that H-theorem for the kinetic equation (9) with the Enskog collisions integral (11) has been proved by P.Résibois [15, 16].

In the same paper [6] another case was also considered, when the interparticle interaction potential is modelled as a sum of a short-range potential (hard spheres, for example) and some long-range smooth potential;

$$\Phi = \begin{cases} \Phi^{hs}, & |\boldsymbol{r}| < \sigma^*; \\ \Phi^{l}, & |\boldsymbol{r}| \geq \sigma^*; \end{cases}$$

where σ^* is the effective diameter of hard spheres, which depends on the method of splitting the potential $\Phi(|\boldsymbol{r}|)$ into short- and long-range parts.

If the time retarding and spatial inhomogeneity will be neglected, then we can present collision integral (10) in the second approximation with respect to interacting potential $\Phi(|\boldsymbol{r}|)$ as follows [13]:

$$I_{col}(x_1; t) = I_{col}^{hs}(x_1; t) + I_{col}^{mf}(x_1; t) + I_{col}^{l}(x_1; t), \tag{12}$$

$$I_{col}^{hs}(x_1; t) = \int dx_2 \, \hat{T}^{hs}(1,2) g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t) \, f_1(x_1; t) \, f_1(x_2; t), \tag{13}$$

$$I_{col}^{mf}(x_1; t) = \int dx_2 \, \imath L^{l}(1,2) g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t) \, f_1(x_1; t) \, f_1(x_2; t), \tag{14}$$

$$I_{col}^{l}(x_1; t) = \tag{15}$$
$$\frac{1}{m} \frac{\partial}{\partial v_{1,\alpha}} \int d\boldsymbol{g} \; J_{\alpha\beta}(\boldsymbol{g}) \left(\frac{\partial}{\partial v_{1,\beta}} - \frac{\partial}{\partial v_{2,\beta}} \right) f_1(x_1; t) \, f_1(\boldsymbol{r}_1 + \boldsymbol{r}_{12}, \boldsymbol{v}_2; t),$$

where

$$J_{\alpha\beta}(\boldsymbol{g}) = \tag{16}$$
$$\frac{1}{m} \int\limits_{\sigma^*}^{\infty} dr_{12} \int d\hat{\sigma} \; r_{12}^2 g_2(\boldsymbol{r}_1, \boldsymbol{r}_1 + \boldsymbol{r}_{12}; t) \left[\frac{\partial \Phi^l(|\boldsymbol{r}_{12}|)}{\partial r_{12,\alpha}} \right] \int\limits_{-\infty}^{t} d\tau \left[\frac{\partial \Phi^l(|\boldsymbol{r}_{12} + \boldsymbol{g}\tau|)}{\partial r_{12,\beta}} \right],$$

and

$$\boldsymbol{g} = \boldsymbol{v}_2 - \boldsymbol{v}_1, \qquad \hat{\sigma} = (\boldsymbol{r}_2 - \boldsymbol{r}_1)/|\boldsymbol{r}_2 - \boldsymbol{r}_1|.$$

The first term in the right hand part of (12) is the Enskog collision integral (13), where $g_2(\boldsymbol{r}_1, \boldsymbol{r}_2; t)$ is the quasi-equilibrium pair distribution function for system of particles with

and inverse temperature $\beta(\boldsymbol{r};t)$ functionally. The second term (14) is the mean field influence, and the third one (15) is written in the form of Landau-like collision integral. If $\Phi^l(|\boldsymbol{r}|) = \dfrac{(Ze)^2}{r}$ is the Coulomb potential, equation (12) with structure of (16) is a generalization of Landau collision integral without divergency for short-range distances between particles (that is different from the usual Landau collision integral [3, 20] because short-range interactions are taken into account correctly). Therefore we can call such kinetic equation (9) with collision integral (12) as the Enskog-Landau kinetic equation for the system of charged hard spheres. It is necessary to emphasize that in equation (16) the long-range divergency still remains. To avoid this problem sequentially we have to consider kinetic equation with taking into account of dynamical screening effects [2, 3]. But this way is impossible in Enskog-Landau kinetic equation. Only one we can do for further calculation is to change upper integral limit to some finite value, which could have a meaning of value of statical screening in our system (see below). solve this problem we must consider dynamical screening effects.

Following [13], we can write for $I_{col}^{hs}(x_1;t)$ the next representation:

$$I_{col}^{hs}(x_1;t) = I_{col}^{hs(0)}(x_1;t) + I_{col}^{hs(1)}(x_1;t), \tag{17}$$

$$I_{col}^{hs(0)}(x_1;t) = \int d\boldsymbol{v}_2\, d\varepsilon\, b\, db\, g\, g_2 \Big(\sigma^+|n(\boldsymbol{r};t),\beta(\boldsymbol{r};t)\Big) \times$$
$$\times \Big(f_1(\boldsymbol{r}_1,\boldsymbol{v}_1';t)\, f_1(\boldsymbol{r}_1,\boldsymbol{v}_2';t) - f_1(\boldsymbol{r}_1,\boldsymbol{v}_1;t)\, f_1(\boldsymbol{r}_1,\boldsymbol{v}_2;t)\Big), \tag{18}$$

$$I_{col}^{hs(1)}(x_1;t) = \sigma^3 \int d\hat{\boldsymbol{r}}_{12}\, d\boldsymbol{v}_2\ (\hat{\boldsymbol{r}}_{12}\boldsymbol{g})\,\Theta(\hat{\boldsymbol{r}}_{12}\boldsymbol{g})\,\hat{\boldsymbol{r}}_{12} g_2 \Big(\sigma^+|n(\boldsymbol{r};t),\beta(\boldsymbol{r};t)\Big) \times$$
$$\times \Big(f_1(\boldsymbol{r}_1,\boldsymbol{v}_1';t)\,\nabla f_1(\boldsymbol{r}_1,\boldsymbol{v}_2';t) + f_1(\boldsymbol{r}_1,\boldsymbol{v}_1;t)\,\nabla f_1(\boldsymbol{r}_1,\boldsymbol{v}_2;t)\Big). \tag{19}$$

Here we use definitions: ε is an azimuthal angle of scattering, $\hat{\boldsymbol{r}}_{12}$ is the unit vector, $\boldsymbol{g} = \boldsymbol{v}_2 - \boldsymbol{v}_1$, b is the impact parameter, $\Theta(x)$ is the unit function, $\boldsymbol{v}_1' = \boldsymbol{v}_1 + \hat{\boldsymbol{r}}_{12}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})$, $\boldsymbol{v}_2' = \boldsymbol{v}_2 - \hat{\boldsymbol{r}}_{12}(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{g})$ are velocities of particles after a collision, and σ^+ is a diameter of the particle.

By representing (15) in the cylindrical coordinates, one can introduce the impact parameter b, azimuthal angle of scattering ε, distance along the cylinder axis ξ. Then Boltzmann-like collision integral can be obtained from (15), putting $g_2 \equiv 1$:

$$I_{col}^{l}(x_1;t) = \int d\boldsymbol{v}_2\, d\varepsilon\, b\, db\, g\Big(f_1(\boldsymbol{r}_1,\boldsymbol{v}_1^*;t)\, f_1(\boldsymbol{r}_1,\boldsymbol{v}_2^*;t) - f_1(\boldsymbol{r}_1,\boldsymbol{v}_1;t)\, f_1(\boldsymbol{r}_1,\boldsymbol{v}_2;t)\Big), \tag{20}$$

where

$$\boldsymbol{v}_1^* = \boldsymbol{v}_1 + \Delta\boldsymbol{v}, \qquad \boldsymbol{v}_2^* = \boldsymbol{v}_2 - \Delta\boldsymbol{v},$$
$$\Delta\boldsymbol{v} = -\frac{1}{mg}\int\limits_{-\infty}^{+\infty} d\xi \nabla\Phi^l(|\boldsymbol{r}_{12}|)\Big|_{r_{12}=\sqrt{b^2+\xi^2}}. \tag{21}$$

After these transformations we have the kinetic equation (9), where the collision integral $I_{col}(x_1;t)$ can be presented as the sum of (14), (18), (19) and (20).

We shall solve the equation (9) by means of iterations. Therefore a set of hydrodynamical variables should be introduced for reduced description of the system: hydrodynamical density of mass, momentum and kinetic energy [4, 21]. The conservation laws are to be written down after multiplication of both left and right parts of equation (9) by components of additive invariant vector m, $m\boldsymbol{v}$ as well as by kinetic energy $\frac{mc^2}{2}$ and after integrating over $\boldsymbol{v}$ [4, 21]. The local-equilibrium Maxwell distribution function may be taken as an initial approximation:

$$f_1^{(0)}(x_1;t) = n(\boldsymbol{r}_1;t)\left(\frac{m}{2\pi kT(\boldsymbol{r}_1;t)}\right)^{3/2}\exp\left(-\frac{mc_1^2(\boldsymbol{r}_1;t)}{2kT(\boldsymbol{r}_1;t)}\right), \tag{22}$$

where $n(\boldsymbol{r};t)$ is the density, $\boldsymbol{c}(\boldsymbol{r};t) = \boldsymbol{v} - \boldsymbol{V}(\boldsymbol{r};t)$, and $\boldsymbol{V}(\boldsymbol{r};t)$ is the hydrodynamical velocity. The total distribution function $f_1(x;t)$ has a form $f_1^0(x;t)\Big(1+\varphi(x_1;t)\Big)$ and the correction $\varphi(x_1;t)$ may be expressed through Sonine-Laguerre polynomials [4]. The calculations show [13]:

$$\varphi(x_1;t) = \frac{A(1)}{T(\boldsymbol{r}_1;t)}\sqrt{\frac{m}{2kT(\boldsymbol{r}_1;t)}}\left(\frac{5}{2} - \frac{mc_1^2(\boldsymbol{r}_1;t)}{2kT(\boldsymbol{r}_1;t)}\right)(\boldsymbol{c}_1\cdot\nabla)T(\boldsymbol{r}_1;t) -$$

$$-\frac{mB(0)}{2kT(\boldsymbol{r}_1;t)}\left(\boldsymbol{c}_1\boldsymbol{c}_1 - \frac{1}{3}c_1^2\overleftrightarrow{I}\right) : (\nabla V(\boldsymbol{r}_1;t)) \tag{23}$$

where $\overleftrightarrow{I}$ is the unit tensor; $A(1)$, $B(0)$ are coefficients which satisfy the following relations:

$$A(1) = \frac{15}{8}\sqrt{\frac{\pi}{2}} \quad \times \frac{1+\frac{2}{5}\pi n\sigma^3 g_2(\sigma^+|n,\beta)}{n\Big(g_2(\sigma^+|n,\beta)\,\Omega_{hs}^{(2,2)} + \Omega_l^{(2,2)}\Big)}, \tag{24}$$

$$B(0) = \frac{5}{2}\sqrt{\frac{\pi m}{kT}} \times \frac{1+\frac{4}{15}\pi n\sigma^3 g_2(\sigma^+|n,\beta)}{n\Big(g_2(\sigma^+|n,\beta)\,\Omega_{hs}^{(2,2)} + \Omega_l^{(2,2)}\Big)}. \tag{25}$$

Here, we use notations:

$$\Omega_{hs,l}^{(p,q)} = \int_0^\infty dg_0 g_0^{2q+3}\exp\left(-g_0^2\right)\Omega_{hs,l}^{(p)}, \tag{26}$$

$$\Omega_{hs}^{(p)} = 2\pi\int_0^\sigma bdb\Big(1-\cos^p\chi'(b,g)\Big), \tag{27}$$

$$\Omega_l^{(p)} = 2\pi\int_\sigma^\infty bdb\Big(1-\cos^p\chi^*(b,g)\Big), \tag{28}$$

$$\boldsymbol{g}_0 = \sqrt{\frac{m}{2kT}}\,\boldsymbol{g}.$$

The expressions (26-28) are known as Ω-integrals [4], χ', χ^* being the angles of scattering for the hard spheres and Coulomb particles respectively.

The Ω-integrals can be calculated exactly [4] or approximately, if the first way fails. We shall attempt to do this from the geometrical point of view. Considering the dynamics of collision of hard spheres, we have [4]

$$\cos\frac{\chi'}{2} = \frac{b}{2}, \qquad \text{and} \qquad \Omega_{hs}^{(2,2)} = 2\pi\sigma^2. \tag{29}$$

imation can be obtained [22]:

$$\sin\chi^* \approx 2\frac{(Ze)^2}{mg_0^2}\int\limits_0^\infty \frac{d\xi}{(b^2+\xi^2)^{\frac{3}{2}}}. \tag{30}$$

This expression for $\sin\chi^*$ leads to logarithmical divergency in $\Omega_l^{(2,2)}$ at integration over infinite sight parameter (28). To avoid this difficulty, the infinite sight parameter in the integral (28) for calculating $\Omega_l^{(2,2)}$ should be replaced by the Debye-type finite radius D of screening for such system. Then we obtain:

$$\Omega_l^{(2,2)} = \pi^3\frac{(Ze)^4}{(kT)^2}\ln\frac{D}{\sigma}. \tag{31}$$

The stress tensor and heat flux vector for obtained distribution function $f_1(x_1;t)$ in the first approximation are:

$$\overleftrightarrow{P}\,(\boldsymbol{r}_1;t) = P\,(\boldsymbol{r}_1;t)\,\overleftrightarrow{I}\,-æ\Big(\nabla V\,(\boldsymbol{r}_1;t)\,\Big) - 2\eta\,\overleftrightarrow{S}\,(\boldsymbol{r}_1;t)\,, \tag{32}$$

where P is the pressure, $\overleftrightarrow{S}\,(\boldsymbol{r}_1;t)$ is the shift tensor of velocities, æ is the coefficient of bulk viscosity

$$æ = \frac{4}{9}\sigma^4 n^2 g_2\left(\sigma^+|n,\beta\right)\sqrt{\pi mkT}, \tag{33}$$

η is the coefficient of shear viscosity

$$\eta = \frac{3}{5}æ + \frac{1}{2}nkT\left(1+\frac{4}{15}\pi n\sigma^3 g_2\left(\sigma^+|n,\beta\right)\right)B\,(0)\,. \tag{34}$$

The heat flux vector has the form

$$\boldsymbol{q}(\boldsymbol{r}_1;t) = -\lambda\,(\nabla\cdot T(\boldsymbol{r}_1;t))\,, \tag{35}$$

where λ is the thermal conductivity:

$$\lambda = \frac{3k}{2m}æ + \frac{5}{4}nk\sqrt{\frac{2kT}{m}}\left(1+\frac{2}{5}\pi n\sigma^3 g_2\left(\sigma^+|n,\beta\right)\right)A\,(1)\,. \tag{36}$$

We can consider some particular expressions for the quantities æ, η and λ. If $n\to 0$, then æ, η, and λ coincide with respective quantities obtained by solving Boltzmann's equation for low density systems of point-like charged particles [21]. If $Z=0$, then we obtain the results of RET-theory [14].

4 Numerical calculations

The numerical calculations were carried out for the viscosities æ (33), η (34) and thermal conductivity λ (36), where the dense once-ionized argon was chosen as a dense one-component plasma in a homogeneous neutralizing continuum. In the case $Z=0$, the obtained results were compared with [23]. In these papers dense neutral argon was considered. Its atoms were modelled as hard spheres, and obtained results are in a good agreement with [23] if the theory parameter σ is chosen correctly.

The binary correlation distribution function $g_2(\sigma^+|n,\beta)$ was taken from [24] where it is presented as the functional of σ and n, but not of β:

$$g_2\Big(\sigma,n\Big) = \left(1-\frac{\pi}{12}n\sigma^3\right)\times\left(1-\frac{\pi}{6}n\sigma^3\right)^{-3}. \tag{37}$$

can give for high densities incorrect values being less than σ. Therefore it is presented in the form, that was proposed in [25], where the hard spheres diameter is also taken into account:

$$D = \sigma \frac{1-\Delta}{1+2\Delta} \left[\sqrt{1 + 4Ze\sigma \left(\frac{1-\Delta}{1+2\Delta} \right)^2 \sqrt{\frac{\pi n}{kT}}} - 1 \right]^{-1}, \tag{38}$$

where

$$\Delta = \frac{1}{6} \pi n \sigma^3.$$

Such substitution of σ, D and $g_2(\sigma, n)$ allows to obtain transport coefficients as functions of density n and temperature T. The behaviours of æ, η and λ were investigated for various values of n and T including regions mentioned in [23], but only for the case giving value of D greater than σ. The purpose of these calculations was to study the dependence of æ, η and λ on the long-range potential when the density is high. As was expected the behaviour of such transport coefficients in these regions appears to be rather smooth and monotonous. But the small deviation is observed between our results and comparative ones when the long interaction is "switched on". Transport coefficients appear to be slightly sensitive to including the long-range potential. They decrease slowly being of the same order. These behaviour coincide with expectations.

We note that the numerical calculation for æ, η and λ was carried out and compared with data from [26, 27, 28]. As one can see from the figures below, the good coincidence of both data is observed in the "high"-temperature region. Theory parameter σ for different densities was borrowed from [29, 30, 31, 32]. It is possible to improve the results by choosing σ more precise but in general case σ is the function of n, T, Z and finding for this dependence is a microscopic problem.

As it can be seen from the figures below, the most essential deviations between of theoretical calculations from the experimental data arise in the low-temperature region for η (figure 1) and for λ (figure 2). In real physical systems at low temperature one can approach to gas-liquid phase transition point. But our theory is not applicable within critical region near point of phase transition. Except that there is no phase transition in one-component system with only repulsive potential on the contrary to real systems. So our theory works well for description of transport processes at high temperatures far from phase transition point of corresponding real systems.

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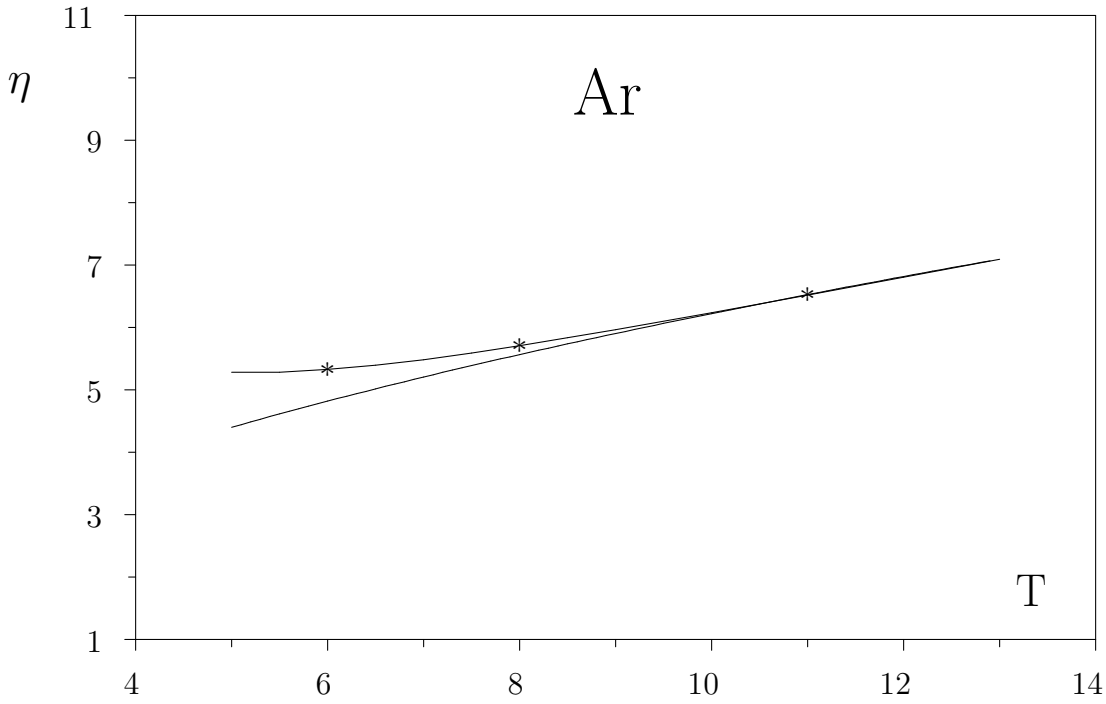

Figure 1: Temperature dependence of shear viscosity η of neutral Ar at $\Delta = 0.1$ ($n = 4.86 \cdot 10^{21}$ cm^{-3}). Solid line represents results from theory, solid marked line represents data of [26, 27]. Both η and T are dimensionless. The transition relations to dimensional data read: $\eta_{dim} = \eta \cdot 10^{-5}\ Pa \cdot sec.$, $T_{dim} = T \cdot 10^2\ K$.

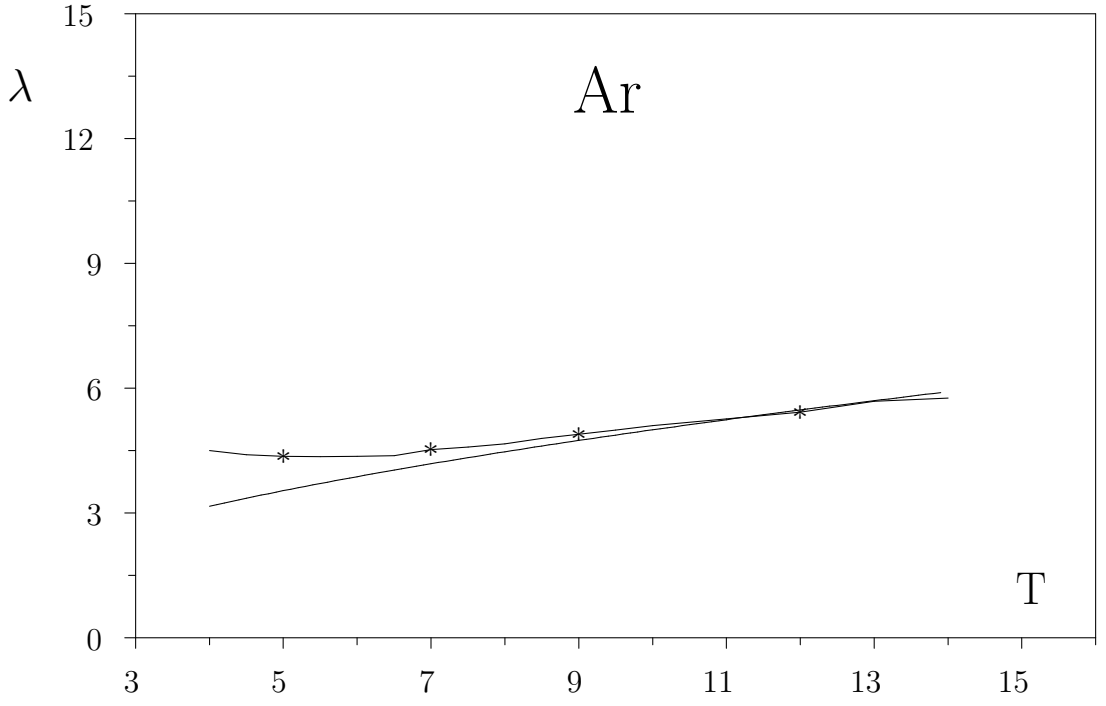

Figure 2: Temperature dependence of thermal conductivity λ of neutral Ar at $\Delta = 0.075$ ($n = 3.644 \cdot 10^{21}$ cm^{-3}). The legend is the same as for figure 1. Both λ and T are dimensionless. The transition relations to dimensional data read: $\lambda_{dim} = \lambda \cdot 10^{-2}\ Wt/(m \cdot K)$, $T_{dim} = T \cdot 10^2\ K$.

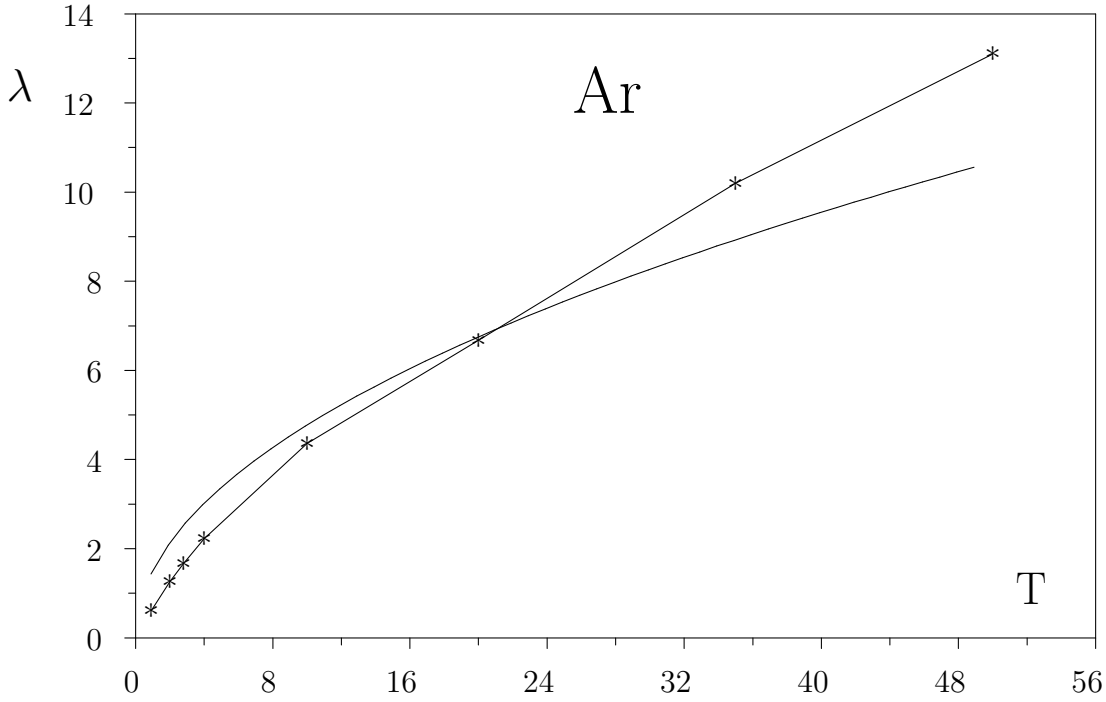

Figure 3: Temperature dependence of thermal conductivity λ of neutral Ar at $\Delta = 0.0125$ ($n = 6.074 \cdot 10^{20}$ cm^{-3}). The legend is the same as for figure 1, but experimental data were taken from [28]. The transition relations to dimensional data read the same as for figure 2.

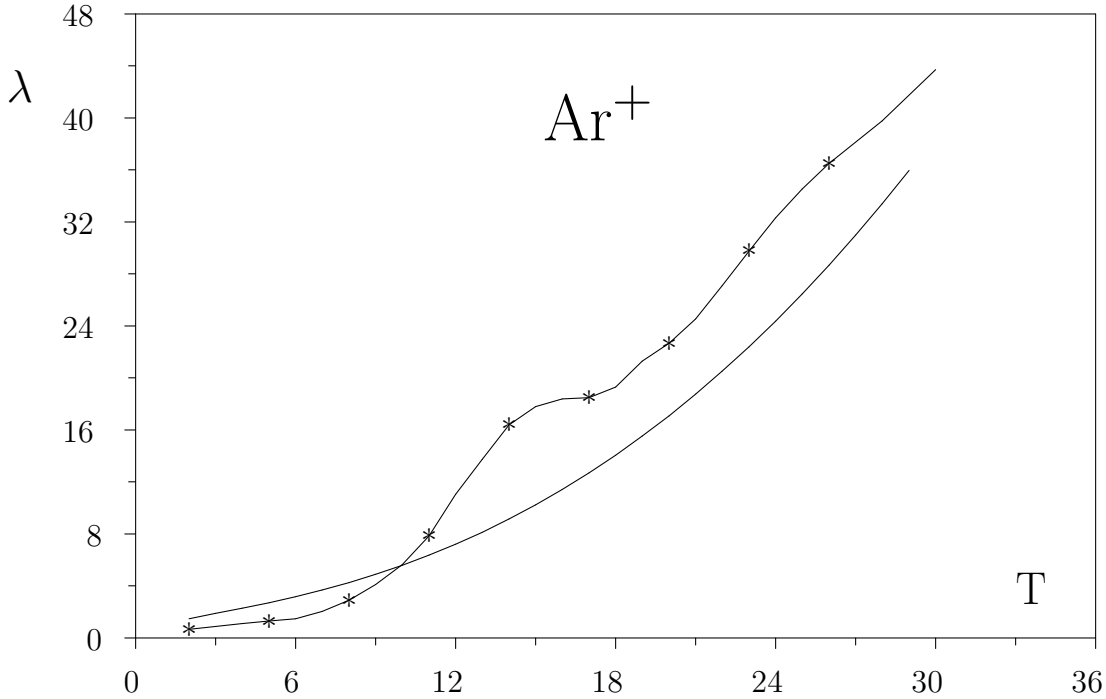

Figure 4: Temperature dependence of thermal conductivity λ of once-ionized Ar at $\Delta = 0.0126$ ($n = 6.123 \cdot 10^{20}$ cm^{-3}). The legend is the same as for figure 1. The transition relations to dimensional data read: $\lambda_{dim} = \lambda \cdot 10^{-1}\ Wt/(m \cdot K)$, $T_{dim} = T \cdot 10^3\ K$.